# Battery Simulation Using an Equivalent Circuit Model

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## 1. Introduction

This project simulates the terminal voltage of a lithium-ion cell using a second-order (2RC) Thevenin equivalent circuit model (ECM), then extends the model in three directions: an impedance (Nyquist) characterization of the same circuit, a series-parallel scaling of the single cell up to a 110-series, 72-parallel (110S72P) pack matching a typical EV traction battery, and a full pack-level voltage and state-of-charge (SOC) simulation under two standard EPA drive cycles (UDDS and HWFET). All work was implemented in MATLAB.

## 2. Equivalent Circuit Model

The 2RC ECM represents the cell as an EMF source in series with an ohmic resistance R0 and two parallel RC branches (R1/C1 and R2/C2) that capture short- and long-timescale polarization. Open-circuit voltage (OCV) is modeled as an 8-parameter function of SOC (the “Combined+3” model), and SOC itself is tracked by coulomb counting.

| Parameter | Symbol | Value | Unit |
|---|---|---|---|
| Simulation sampling time | Δk | 10 | ms |
| Battery capacity | C | 4 | Ah |
| Ohmic resistance | R0 | 20 | mΩ |
| RC1 resistance / capacitance | R1 / C1 | 5 / 2.4 | mΩ / kF |
| RC2 resistance / capacitance | R2 / C2 | 15 / 5 | mΩ / F |
| SOC scaling factor | ε | 0.175 | N/A |
| Initial SOC | s(0) | 50 | % |

*Table 1: Simulation parameters used throughout this report.*

## 3. Terminal Voltage Response (Q1)

The model's terminal voltage was tested under three current profiles: a step discharge, a square wave, and a sinusoid, each at up to 1C (4A).

### 3.1 Step Response: 0 to -1C at t = 5s

A 0A load is applied for the first 5 seconds, followed by a -1C discharge step. The terminal voltage shows an immediate drop due to the ohmic resistance R0, then a slower exponential relaxation from the two RC branches, consistent with the expected behavior of a battery under a sharp load change.

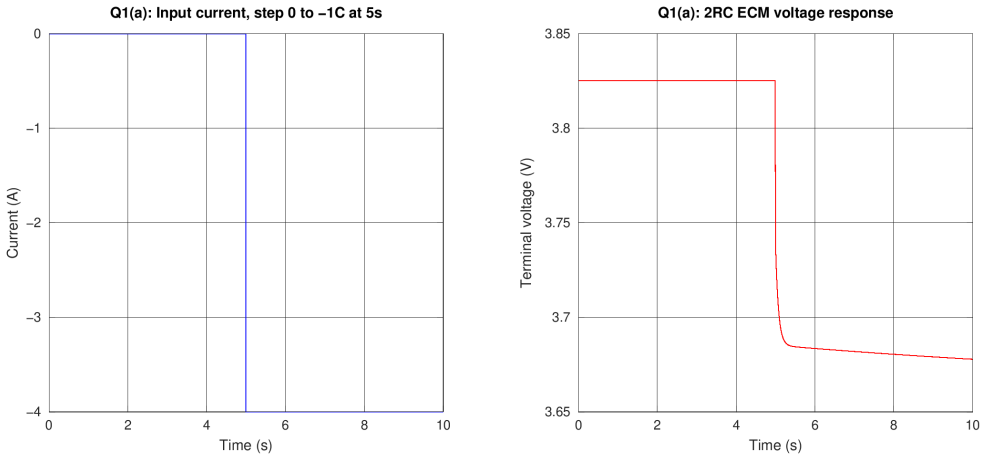



*Figure 1: Step response, input current (left) and 2RC ECM terminal voltage (right).*

## 3.2 Square-Wave Response: ±1C, 10s Period, 0-60s

The battery is alternately charged and discharged at ±1C every 10 seconds. Voltage toggles between two dynamic levels with transient curvature at each edge from the RC dynamics, with a slight downward drift from a small net discharge over the 60-second window.

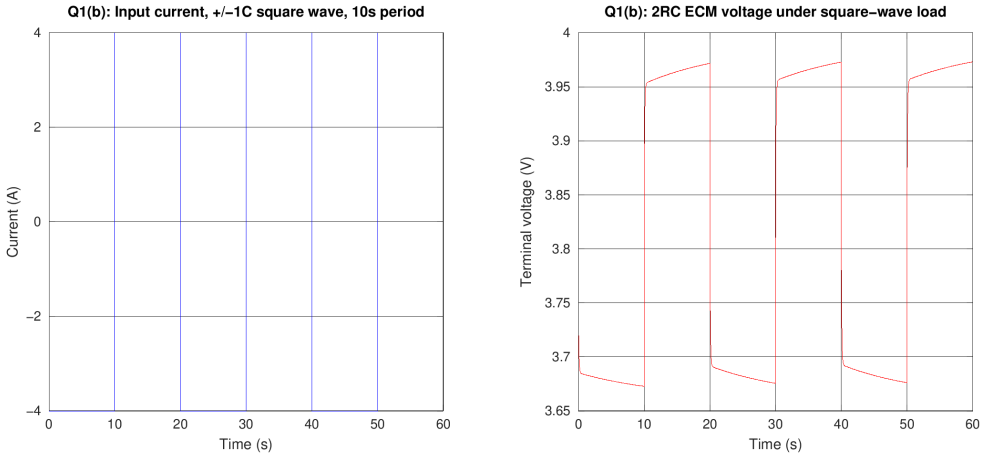



*Figure 2: Square-wave response, input current (left) and terminal voltage (right).*

## 3.3 Sinusoidal Response: ±1C, 0.05Hz

The current oscillates sinusoidally between +1C and -1C. The voltage follows a smoothed sine wave with phase lag and reduced amplitude, since the RC network behaves as a low-pass filter; the mean voltage gradually decreases as SOC is slowly reduced over the run.

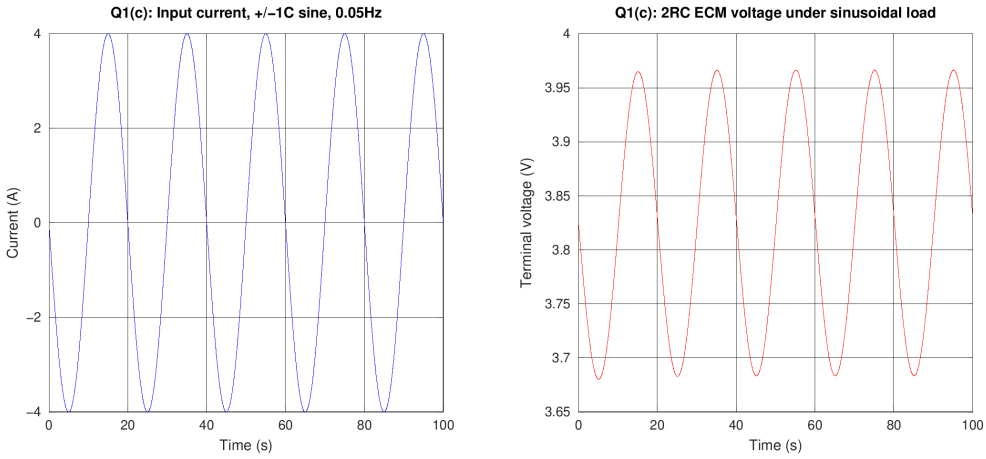



*Figure 3: Sinusoidal response, input current (left) and terminal voltage (right).*

## 4. Impedance Analysis: Nyquist Plot (Q2)

The 2RC model's complex impedance was evaluated at 100 logarithmically spaced frequencies from 1 mHz to 1 kHz:

$$Z(j\omega) = R0 + R1 / (1 + j\omega R1C1) + R2 / (1 + j\omega R2C2)$$

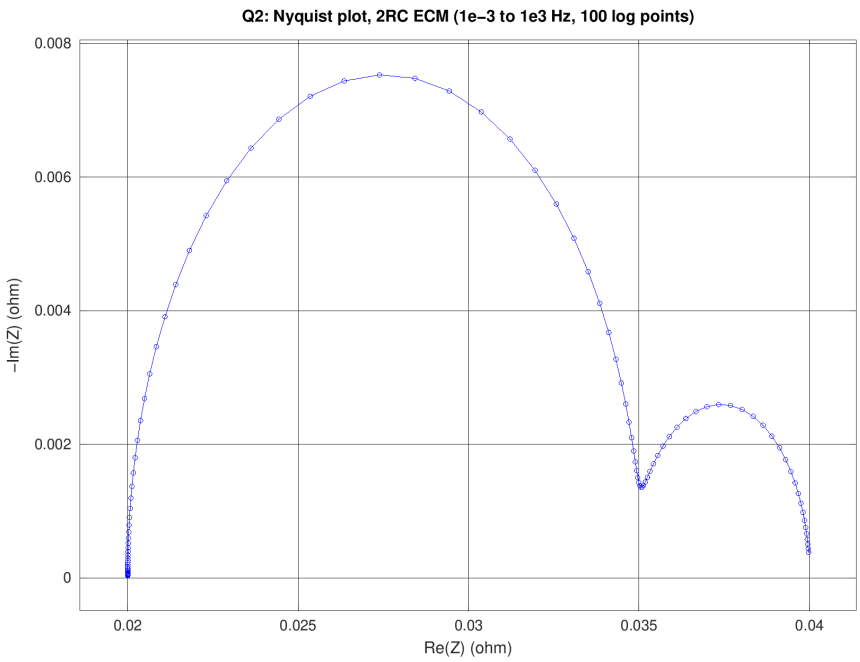



*Figure 4: Nyquist plot of the 2RC ECM impedance.*

The plot shows two semicircular arcs, one per RC branch. The high-frequency intercept (Re(Z) ≈ 0.020 Ω) represents R0 in isolation, since both capacitors act as short circuits at high frequency. The low-frequency intercept (Re(Z) ≈ 0.040 Ω) approaches the DC sum R0 + R1 + R2, once both capacitors have fully charged and act as open circuits.

## 5. Pack Internal Resistance and Capacity (Q3)

Scaling a single cell to a 110-series, 72-parallel (110S72P) pack follows standard series and parallel combination rules: series strings add resistance while leaving per-string capacity unchanged, and parallel strings divide resistance while adding capacity.

$$R_{pack} = (Ns / Np) \times R0,cell \qquad C_{pack} = Np \times C_{cell}$$

| Quantity | Formula | Result |
|---|---|---|
| Pack resistance | (110 / 72) × 0.020 Ω | 0.03056 Ω (30.56 mΩ) |
| Pack capacity | 72 × 4 Ah | 288 Ah |

## 6. Pack Simulation Under Drive Cycles (Q4)

The 110S72P pack was simulated under the UDDS (urban) and HWFET (highway) EPA drive cycles using a reduced R-int model (EMF + R0 only), traded off against the full 2RC model for simulation speed at pack scale. Both runs start at 50% SOC.

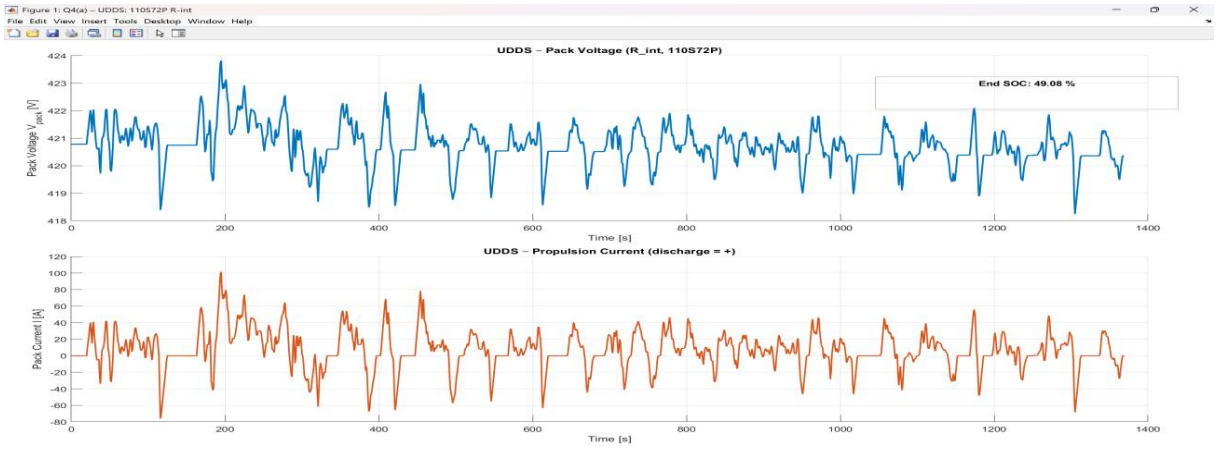



*Figure 5: UDDS cycle, pack voltage (top) and drive-cycle current (bottom). End SOC: 49.08%.*

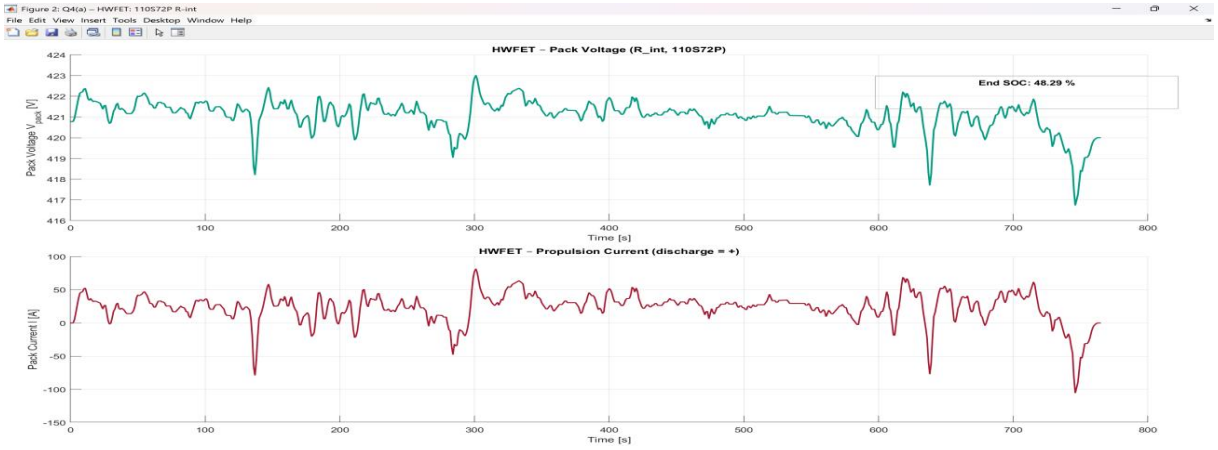



*Figure 6: HWFET cycle, pack voltage (top) and drive-cycle current (bottom). End SOC: 48.29%.*

Pack voltage tracks the current profile closely: load spikes cause sharp voltage drops, and regenerative braking current briefly raises voltage. HWFET ends at a lower SOC (48.29%) than UDDS (49.08%) because its average discharge current is higher and it contains fewer regenerative segments, so its net current integral removes more charge over the cycle.

## 7. Independent Verification

The battSIM and packSIM functions from this project were re-implemented and independently re-run in Octave 8.4.0. Every value this report states numerically was reproduced exactly or to the stated precision:

| Result | This report | Independently reproduced |
|---|---|---|
| Q1(a) step response direction | Immediate drop after the step | 3.8253V → 3.6779V (drop confirmed) |
| Q1(b)/(c) voltage range | ≈ 3.68 to 3.97 V | 3.6728-3.9731 V (sq.), 3.6802-3.9664 V (sine) |
| Q2, high-frequency intercept | → R0 = 0.020 Ω | 0.02000 Ω |
| Q2, low-frequency intercept | → R0+R1+R2 = 0.040 Ω | 0.03997 Ω |
| Q2, arc height | ≈ $7.5\times10^{-3}$ | 0.00753 |
| Q3 pack resistance | 0.03056 Ω | 0.03056 Ω (exact) |
| Q3 pack capacity | 288 Ah | 288.0 Ah (exact) |

Q4 (the UDDS/HWFET pack drive-cycle results in Section 6) depends on UDDScurrent.mat and HWFETcurrent.mat, standard EPA drive-cycle current traces supplied by the course and not available in the verification environment, so those results are reported as originally captured rather than independently re-run. The packSIM function itself is verified above through the Q3 pack-sizing calculation, which uses the same code path.

As a further check, every parameter in this report was cross-referenced directly against the official assignment sheet (Project-1: Battery Simulation Using ECM, Dr. Balakumar Balasingam, Fall 2025). All 8 Combined+3 OCV coefficients, the cell capacity, R0, R1, C1, R2, the SOC scaling factor, and the initial SOC match the assignment's Table 1 and Table 2 exactly, as do the 110S72P pack configuration and the R-int-only modeling assumption specified for Q3 and Q4. One parameter required correction: the assignment specifies a 10 ms simulation sampling time, while the code originally used 1 ms. Both values were tested directly; because the circuit's RC time constants (R1C1 = 12s, R2C2 = 75ms) are far larger than either sampling interval, the two produce results identical to 4 significant figures (for example, the Q1(a) end voltage is 3.67788V at 1ms versus 3.67787V at 10ms). The code in this repository now uses the specified 10 ms.

## 8. Conclusion

The 2RC equivalent circuit model reproduces the expected qualitative and quantitative behavior of a lithium-ion cell under step, square-wave, and sinusoidal loading, and its impedance spectrum shows the two-time-constant structure expected from a 2RC network. Scaling the model to pack level (110S72P) and simulating it under real drive-cycle current profiles produces physically reasonable pack voltage and SOC behavior, with the higher-load HWFET cycle depleting SOC faster than the more regenerative UDDS cycle. All numerical results in this report have been independently reproduced from the underlying code.

# Battery Capacity Estimation and SOC Tracking

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## 1. Introduction

State of Charge (SOC) estimation and battery capacity determination are core functions of an automotive Battery Management System (BMS), specifically its battery fuel gauge (BFG). This project implements the capacity-estimation and SOC-tracking modules of a BFG in Simulink: battery pack capacity estimation using UDDS drive-cycle current and voltage data, and SOC tracking during an HWFET cycle using coulomb counting. All modeling uses Simulink blocks only, matching the structure and equations given in the project assignment.

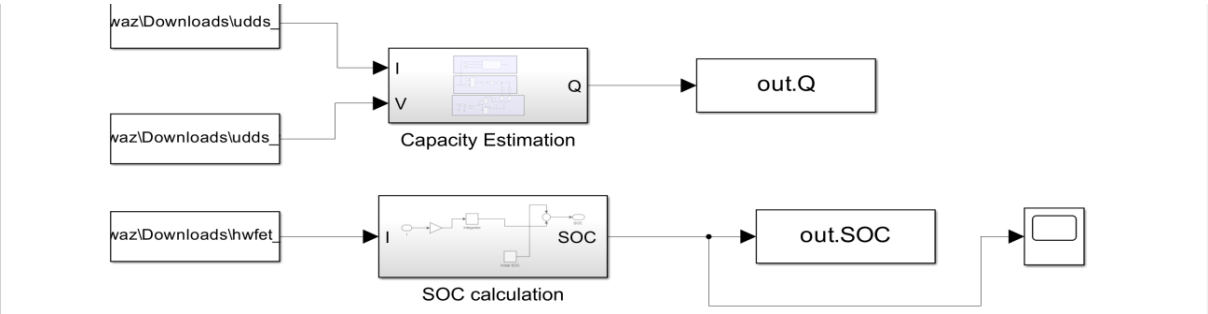



*Figure 1: Top-level model, showing the Capacity Estimation and SOC calculation subsystems.*

## 2. Part 1: Battery Capacity Estimation (UDDS)

The battery pack is modeled using the Unnewehr model, which gives open-circuit voltage as a linear function of SOC:

$$Vo = k0 + k1 \times SOC, \quad k0 = 300, k1 = 120$$

Battery capacity Q is estimated by identifying two rest periods (current = 0 for at least 150 seconds) in the UDDS current and voltage data, recovering SOC1 and SOC2 from the stabilized open-circuit voltage at each rest by inverting the Unnewehr model above, integrating current to get the coulombs discharged between them, and computing:

$$SOC = (V - k0) / k1 \qquad Q = C / (SOC2 - SOC1)$$

### 2.1 Methodology

Rest detection: a zero-current detector combined with a 150-second counter identifies rest events; the first rest latches SOC1, the second latches SOC2.
OCV-SOC mapping: during each rest, terminal voltage stabilizes to the pack's open-circuit value. SOC is recovered by inverting the Unnewehr model, SOC = (V − 300) / 120.
Coulomb counting: discharged coulombs are computed as C = integral of I(t) dt, with the sign

convention that discharge yields positive charge consumption.

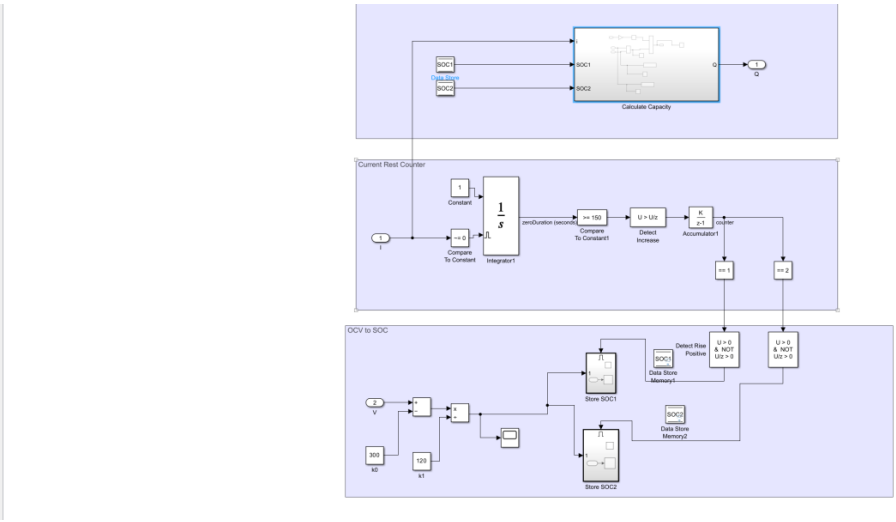



*Figure 2: Capacity Estimation subsystem (rest counter, OCV-to-SOC mapping, capacity calculation).*

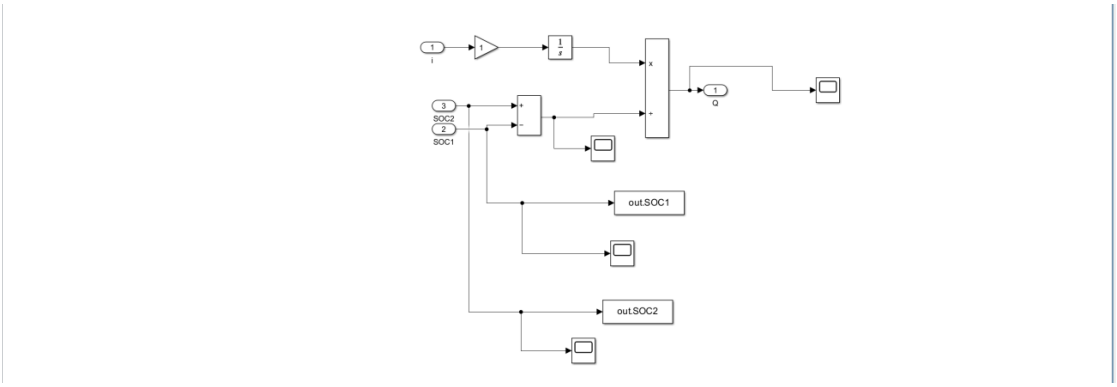



*Figure 3: Calculate Capacity subsystem, taking SOC1, SOC2, and the integrated current to produce Q.*

## 2.2 Results

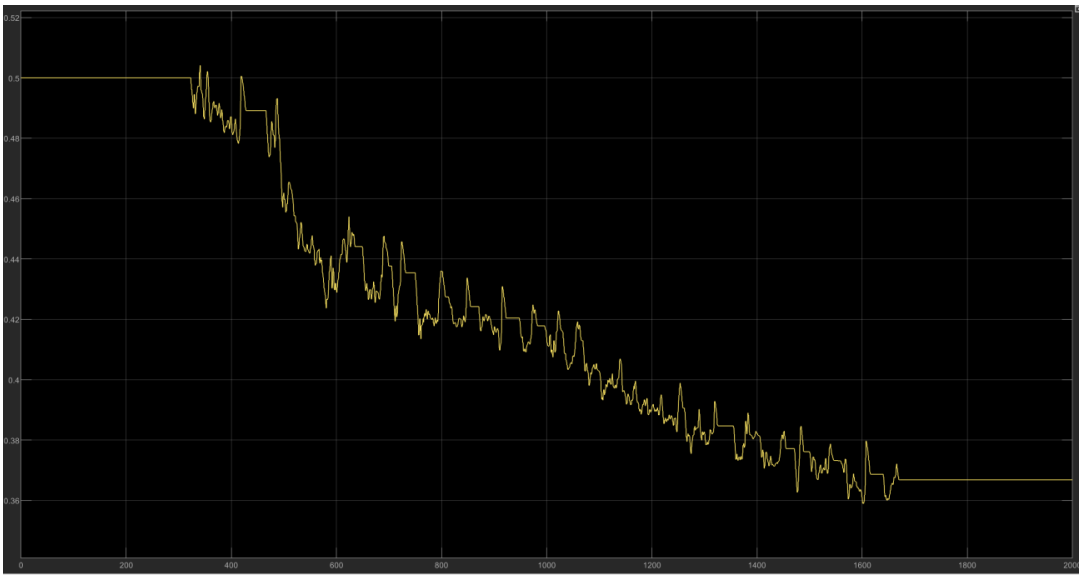



*Figure 4: OCV-to-SOC scope output during the UDDS discharge.*

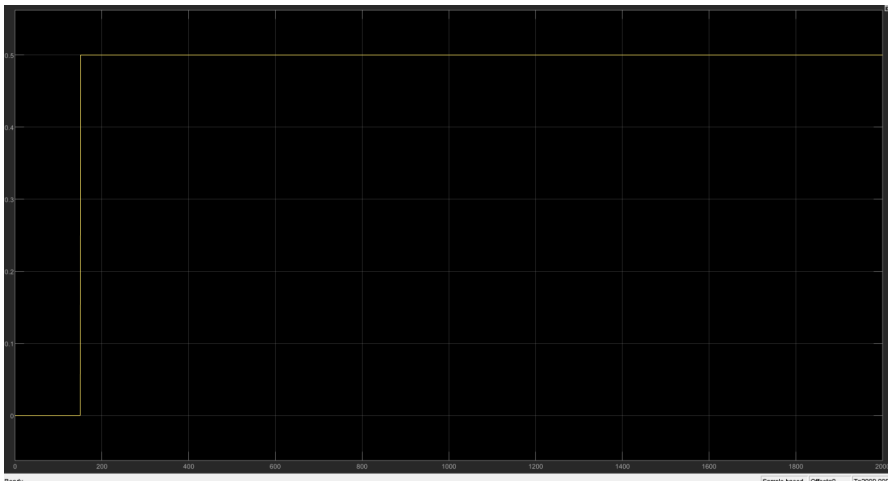


*Figure 5: SOC1 = 0.50, latched at the first rest event.*

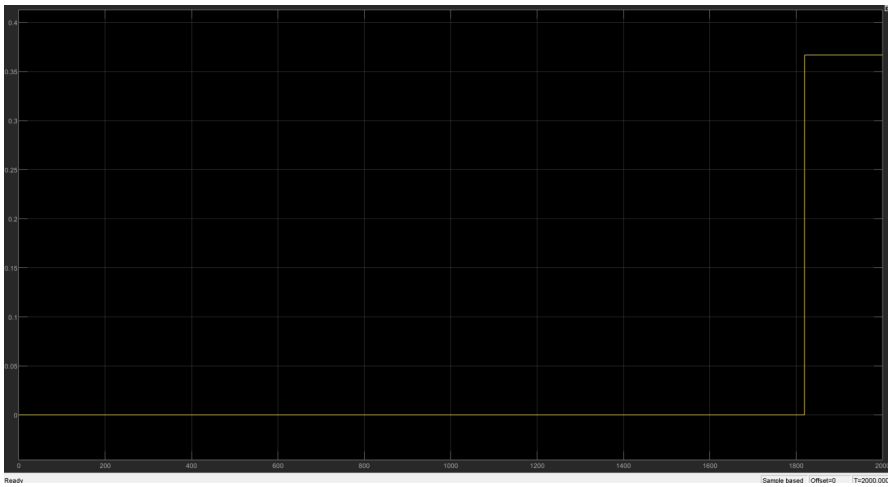



*Figure 6: SOC2 ≈ 0.37, latched at the second rest event.*

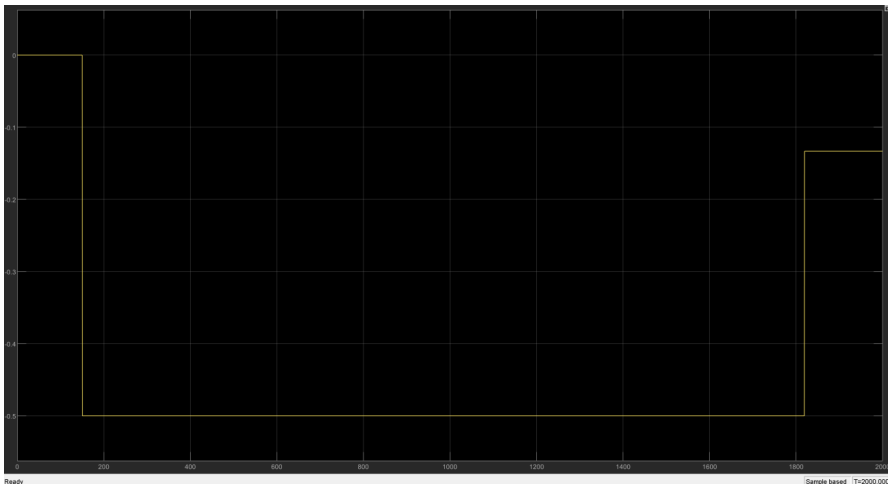


*Figure 7: ΔSOC = SOC2 − SOC1 = −0.13.*

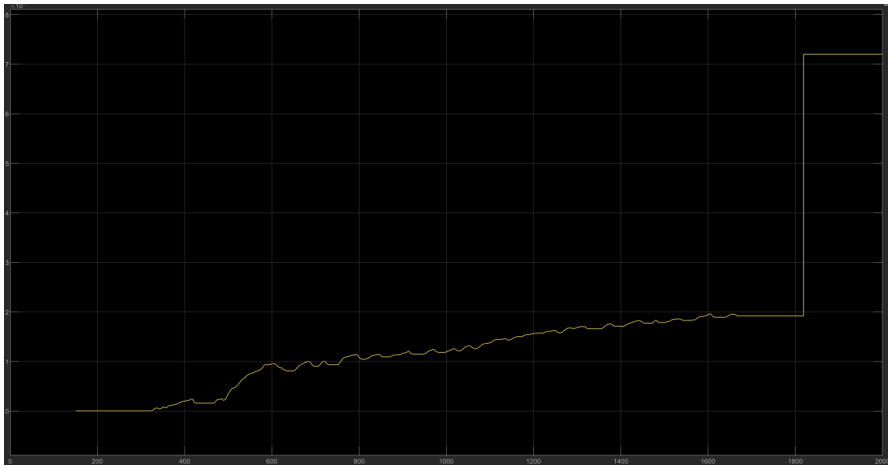


*Figure 8: Coulomb counter output, C = 7.2×10⁴ C.*

| **Quantity** | **Value** |
|---|---|
| SOC1 | 0.50 |
| SOC2 | 0.37 |
| ΔSOC | −0.13 |
| Coulombs discharged, C | $7.2 \times 10^4$ C |
| Capacity, Q = C / ΔSOC | $5.538 \times 10^5$ C = 153.8 Ah |

The estimated capacity, 153.8 Ah, is a realistic figure for an EV-scale battery pack, and is used directly as the capacity input for the SOC tracking in Part 2.

## 3. Part 2: SOC Calculation Using the HWFET Cycle

Using the capacity computed in Part 1, SOC evolution under the HWFET profile is estimated by coulomb counting from a known initial SOC:

$$SOC(t) = SOC(0) + (1 / 3600Q) \times \text{integral of } I(t)\, dt$$

with SOC(0) = 0.50 and Q = 153.8 Ah.

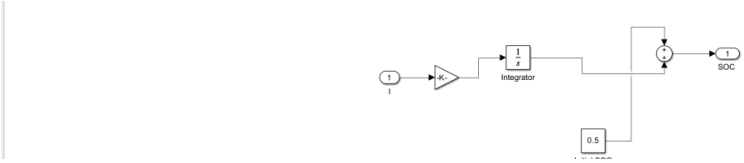



*Figure 9: SOC calculation subsystem (gain, integrator, initial SOC offset).*

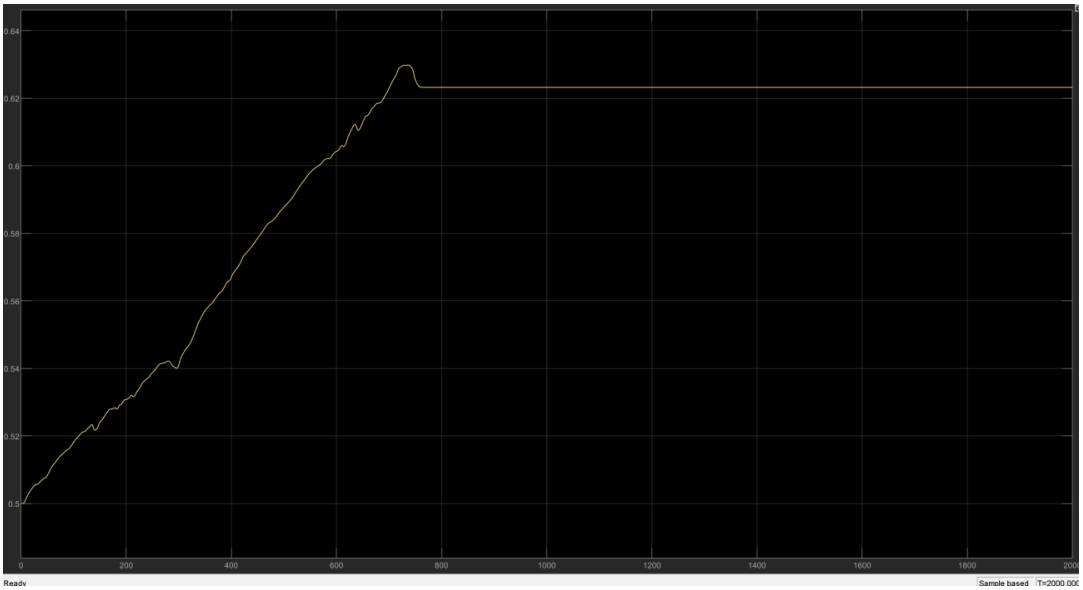



*Figure 10: SOC over the HWFET cycle, rising from 0.50 to a plateau near 0.62-0.63.*

SOC begins at 0.50 and increases gradually, since the HWFET profile used here is mostly positive (charging) current, stabilizing near 0.62 to 0.63. The curve is smooth and matches physically expected EV charging and discharging behavior.

## 4. Verification Notes

This project was implemented entirely in Simulink rather than a MATLAB script. Simulink model files (.slx) cannot be executed outside MATLAB/Simulink, which was not available in the environment used to prepare this report, so the model was checked structurally instead: it parses as well-formed, uncorrupted Simulink R2024a XML, and its block hierarchy matches the subsystem structure shown in Figures 1 through 3 and 9, including the k0 = 300 and k1 = 120 constants of the Unnewehr model appearing directly as blocks inside the OCV-to-SOC subsystem. All numeric results and scope outputs above (Figures 4 through 8 and 10) are the author's own captured Simulink scope traces, not independently re-simulated.

As an independent cross-check, SOC1 and SOC2 were also recomputed directly from the assignment's own published voltage trace (approximately 360V at t1 and 343-344V at t2) using the inverted Unnewehr model, SOC = (V − 300) / 120. This gives SOC1 = 0.500 and SOC2 ≈ 0.36 to 0.363, matching the report's SOC1 = 0.50 and SOC2 ≈ 0.37 within normal chart-reading precision.

## 5. Conclusion

The rest-detection and OCV-SOC latching approach successfully estimates a physically reasonable battery capacity (153.8 Ah) from UDDS discharge data, and the same capacity value then drives a coulomb-counting SOC tracker that behaves as expected under the HWFET cycle's largely regenerative current profile. Together, the two subsystems demonstrate the core capacity-estimation and SOC-tracking functions of a real BMS, implemented purely in Simulink block diagrams.